SIMULATION OF MECHANICAL TESTS OF COMPOSITE MATERIAL USING ANISOTROPIC HYPERELASTIC CONSTITUTIVE MODELS

Tomáš Lasota*, Jiří Burša*

This paper deals with experiments and computational simulations of composite material with hyperelastic matrix and steel fibres; the main goal is to compare simulations, performed at various levels of computational models, and experiments on the fibre composite material with hyperelastic matrix. For this purpose tension and bending tests and their computational simulations were carried out. Two different levels of computational models – bimaterial and unimaterial ones – were used for simulations. Results of experiments and simulations for different declinations of fibres are presented.

Keywords : *anisotropy, hyperelasticity, Cosserat elasticity, composite material, fibre-reinforcement*

1. Introduction

The theory of finite deformations of elastic materials reinforced by fibres was founded by Adkins and Rivlin [1]. Their theory described an isotropic elastic material with no extensibility in the direction of fibres and they assumed that the reinforcing fibres lay in discrete surfaces. A different approach was established by Spencer [2]. In this theory the fibre direction is characterized by a unit vector in the reference configuration.

The fibre vector formulation has been applied to many kinds of material behaviour. Particular applications of the theory of finite elastic deformations are in Spencer [2, 3] and Rivlin [4]. Presently, this theory [2] is used in various kinds of applications of composite materials, either in industry or in composite biomaterials. Concerning examples of industrial use, readers are referred e.g. to [5], where authors simulated response of air-spring (rubber matrix and textile cords) used for inhibition of vibrations of driver's seat. On the other side, arterial walls represent characteristic examples of composite biomaterials. The arterial wall is composed mainly of isotropic matrix material (elastin) and two families of fibres (collagen). A multi-layer model for arterial wall was proposed by Holzapfel [6].

This paper deals with experiments and computational simulations of composite material with elastomer matrix and steel fibres. The computational simulations were realized at various levels of computational models and results were compared with experiments. Two different model levels were used for simulations, namely bimaterial and unimaterial computational models. In the bimaterial model, both matrix and fibres 3D geometries were created. Then material properties of the matrix were prescribed by a hyperelastic constitutive model and for properties of steel fibres a linear elastic constitutive model was used.

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In the unimaterial computational model, however, only a geometric model of the whole composite body was created, without distinguishing between matrix and fibres. Material properties of the whole composite material specimen were prescribed as homogeneous by an anisotropic hyperelastic constitutive model.

The main aim of this paper is to ascertain whether the unimaterial computational model with polynomial form of strain energy density function can be used instead of the bimaterial model; the unimaterial model has got several advantages in comparison with the bimaterial model – a mapped mesh with lower number of elements can be used, low order computational time, simple creation of the model geometry (fibres are not created, but they are involved in the anisotropic constitutive model).

2. Experiment

Two types of experiments were carried out – uniaxial tension test and bending test. Specimens with different declinations of fibres (0°, 15°, 45°, 60° and 90°) and with dimensions 125×25×2.9 mm were used for both tests. Diameter of the fibres was 0.45 mm.

Tension tests were realized using ZWICK universal testing machine. Each specimen was clamped in grips (Fig. 1) and loaded by uniaxial tension. Deformation in the middle part of the specimen was recorded by extensometers; the distance between extensometer levers was 20 mm.

All the specimens were preconditioned before testing, due to eliminate Mullin's effect [11]. For this purpose, the specimen was loaded by 5mm total elongation and then it was unloaded immediately. This loading cycle was repeated five times; differences between the last two of these five cycles were not more substantial.

Three preconditioned specimens were prepared with the same declination of fibres, i.e. each tension test was repeated three times for all of the above declinations of fibres; thus, 15 tests have been carried out in total. The dependency between the force and the elongation of the middle part of the specimen was recorded in these experiments.

To get comprehensive information about deformation of the composite specimen, the displacement field and sequentially strain field in the loaded specimen were evaluated by an optical method (Fig. 2).

Bending tests were realized also with the ZWICK testing machine using a special preparation for three point bending. The specimen (3 pieces for each declination) was placed in the test preparation and pushed against its middle part (Fig. 3). Bending tests were realized

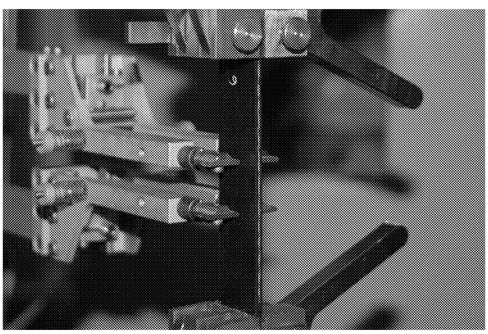

Fig.1: Tension test

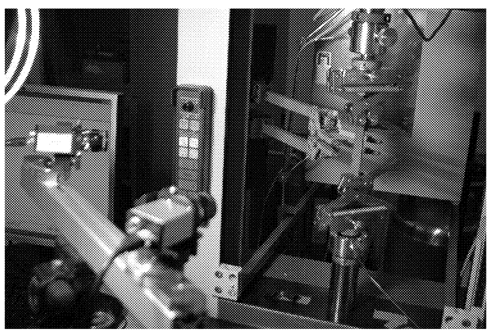

Fig.2: Evaluation of displacement field

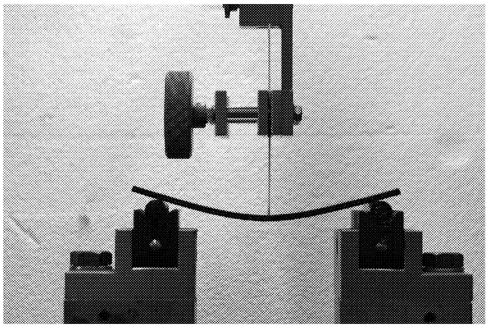

Fig.3: Bending test

on the same specimens, which had been used in the tension tests, i.e. the specimens had been preconditioned by 5 loading cycles in tension. The dependency between the force and the maximal deflection was recorded in these tests.

3. Computational simulations

In computational simulations of the tests, two different model levels have been used. As the first a bimaterial computational model was used, i.e. a 3D model consisting of both elastomer matrix and steel fibres was created. Main disadvantages of this computational model consist in a complicated creation of geometry of fibres (especially if they are not parallel to the specimen edges) and in a rather complex creation of mesh. Due to declination of fibres it is impossible to create a mapped mesh all over the model. Therefore a free mesh with high numbers of elements must be used and herewith the computational time increases substantially. Linear elastic constitutive model was used for fibres, while the behaviour of the elastomer matrix was described by an incompressible hyperelastic model. In the case of tension tests, the 2-parametric Mooney-Rivlin model was used

$$W = c_1\,(I_1 - 3) + c_2\,(I_2 - 3) \;, \tag{1}$$

where I_1 and I_2 represent the first and second modified invariants of the right Cauchy-Green deformation tensor. Whereas the maximal strains in the simulations of the tension tests were about 50 %, in the case of the bending tests the maximal strains were about 5 % only. Hence, the elastomer matrix in the simulations of the bending tests was described by a 3$^{\rm rd}$ order Yeoh model, because this model had achieved the best agreement in simulations of material tests of the elastomer matrix. The form of the used strain energy density function was as folows:

$$W = d_1\,(I_1 - 3) + d_2\,(I_2 - 3)^2 + d_3\,(I_1 - 3)^3 \;, \tag{2}$$

The material parameters c_1, c_2, d_1, d_2 and d_3 were determined in terms of experiments (uniaxial tension test, equibiaxial tension test and planar tension test) with elastomer matrix material by using the least squares method. The specimens for these tests had been also preconditioned in order to eliminate the Mullin's effect. Each elastomer specimen was loaded by the strain of 100 % and then unloaded; this cycle was repeated five times for each specimen. The following material parameters were determined: $c_1 = 0.4727\,$MPa, $c_2 = 0.6992\,$MPa, $d_1 = 4.034\,$MPa, $d_2 = -306.48\,$MPa and $d_3 = 16478\,$MPa.

Common material parameters were used for steel fibres (Poisson's ratio 0.3, Young's modulus 210 GPa). The second model used in simulations of the tests was the unimaterial model, i.e. only a model of the geometry of the whole specimen was created without distinguishing between matrix and fibre materials. The fibre-reinforcement effect was included in the constitutive model by means of a corresponding strain energy density function. A constitutive model with strain energy density function, which includes contributions of both rubber matrix and fibres, was specified in Spencer [2, 3]. This approach is highly advantageous because the fibres need not to be modelled; the specimen is supposed to consist of a homogeneous anisotropic material so that a mapped mesh with a low number of elements can be used and the computational time is low order in comparison with the bimaterial model. In our model the anisotropic incompressible hyperelastic model was used with two different polynomial forms of the strain energy density function:

– in simulations of the tension tests

$$W = c_1 (I_1 - 3) + c_2 (I_2 - 3) + k_1 (I_4 - 1)^2 \; . \tag{3}$$

– in simulations of the bending tests

$$W = d_1 (I_1 - 3) + d_2 (I_2 - 3)^2 + d_3 (I_1 - 3)^3 + k_1 (I_4 - 1)^2 \; . \tag{4}$$

The polynomial form of the strain energy density function is implemented as the only one in ANSYS FEM software. In a general case this strain energy function consists of much more terms with more invariants, and equations (3) or (4) are only its special simplified forms (see [7] for details). Hence, it should be the best established field of application of this specific type of strain energy density function.

The terms with k_1 material parameter in the equations (3) and (4) describe the behaviour of fibres only, while the other terms are related to the pure elastomer matrix.

Invariant I_4 in equations (3) or (4) is defined as follows:

$$I_4 = \mathbf{a}_0 \, \mathbf{C} \, \mathbf{a}_0 = \lambda^2 \; , \tag{5}$$

where $\mathbf{C}$ is the right Cauchy-Green deformation tensor and $\mathbf{a}_0$ is the unit vector of the fibre direction in the reference configuration. Note that the I_4 invariant equals to the square of the λ stretch in the $\mathbf{a}_0$ fibre direction.

The material parametr k_1 in eqs. (3) and (4) was determined on the basis of the following assumption: consider a tension test of composite material, where fibres are oriented in the direction of loading. Then the resulting force in the loading direction is given in particular by stresses in the steel fibres, i.e. stresses in the elastomer matrix are negligible in comparison with stresses in the steel fibres.

When we compare the constitutive equations (1) and (2) in the bimaterial model with constitutive equations (3) and (4) used in the unimaterial model, it is evident that the elastomer matrix is described by the same constitutive equations in both the bimaterial and unimaterial models. The difference occurs in the constitutive description of fibres only, but the material parameter k_1 was determined under condition the stresses in the fibers remain the same in both the unimaterial and bimaterial computational model.

In the case of tension test simulations the specimen was loaded similarly to the experiment. Both ends of each specimen were compressed (similarly to the specimen clamping by the grips in the real experiment) and loaded by a displacement in its longitudinal axis

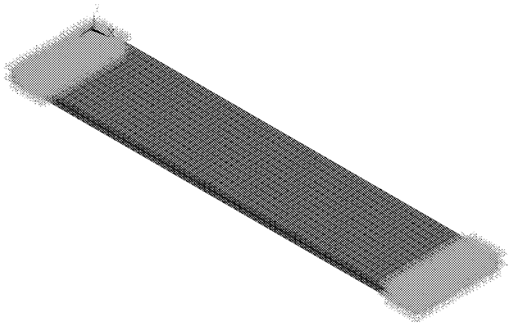

Fig.4: Unimaterial model – tension test

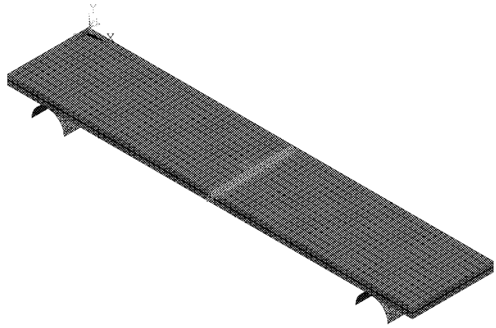

Fig.5: Unimaterial model – bending test

(Fig. 4). Similarly to the experiment, the dependency between the force and the deformation in the middle part of the specimen created the outputs of these simulations.

The unimaterial computational model for simulation of bending test is depicted in Fig. 5. The specimen lies on two rigid cylinders which represent supports in the experiments (Fig. 3). A frictionless contact between them was defined and the specimen deflection was prescribed in its middle part. Similarly to the experiment, the dependency between force and maximum deflection of the specimen was obtained as the result of these simulations.

For illustration, the unimaterial computational model for 15° declination of fibres counts 10 404 elements and the simulation of bending test took approximately 12 hours. On the other hand, the bimaterial computational model of the same test counts 227 404 elements and the simulation of the bending test took over one week with the same computer.

4. Results and discussion

The following five figures (Figs. 6a) to e)) show results from the experimental tension tests and their simulations. From the data obtained in three experiments on specimens with the same declination of fibres, the mean value and confidence interval at the significance level $\alpha = 0.05$ were calculated. In each of the figures below, the same legend is used:

——— Simulation – unimaterial model
– – – Simulation – unimaterial model
⁞ Experiment – mean value and confidence interval ($\alpha = 0.05$)

The results obtained using bimaterial and unimaterial FE models are in very good mutual agreement for any fibre declinations (Fig. 6). Remind that both computational models have the same material models for elastomer matrix and differ only in description of fibres, but material parameter k_1 was proposed so that the stresses in the fibres were the same in both the unimaterial and bimaterial computational models. Hence, both models should give the same results as it was confirmed by these simulations. In terms of these results it is possible to claim that the unimaterial model can be used instead of bimaterial one (in the case of tension load).

However, when we compare every simulation with the corresponding experiment, substantial differences can be found, except for the tension test with zero declination of fibres where simulations are near the upper boundary of the confidence interval. As it was mentioned above, the specimens of the pure elastomer matrix, needed for determine material parameters (c_1, c_2, d_1, d_2, d_3), were preconditioned by five loaded cycles with 100 % strain

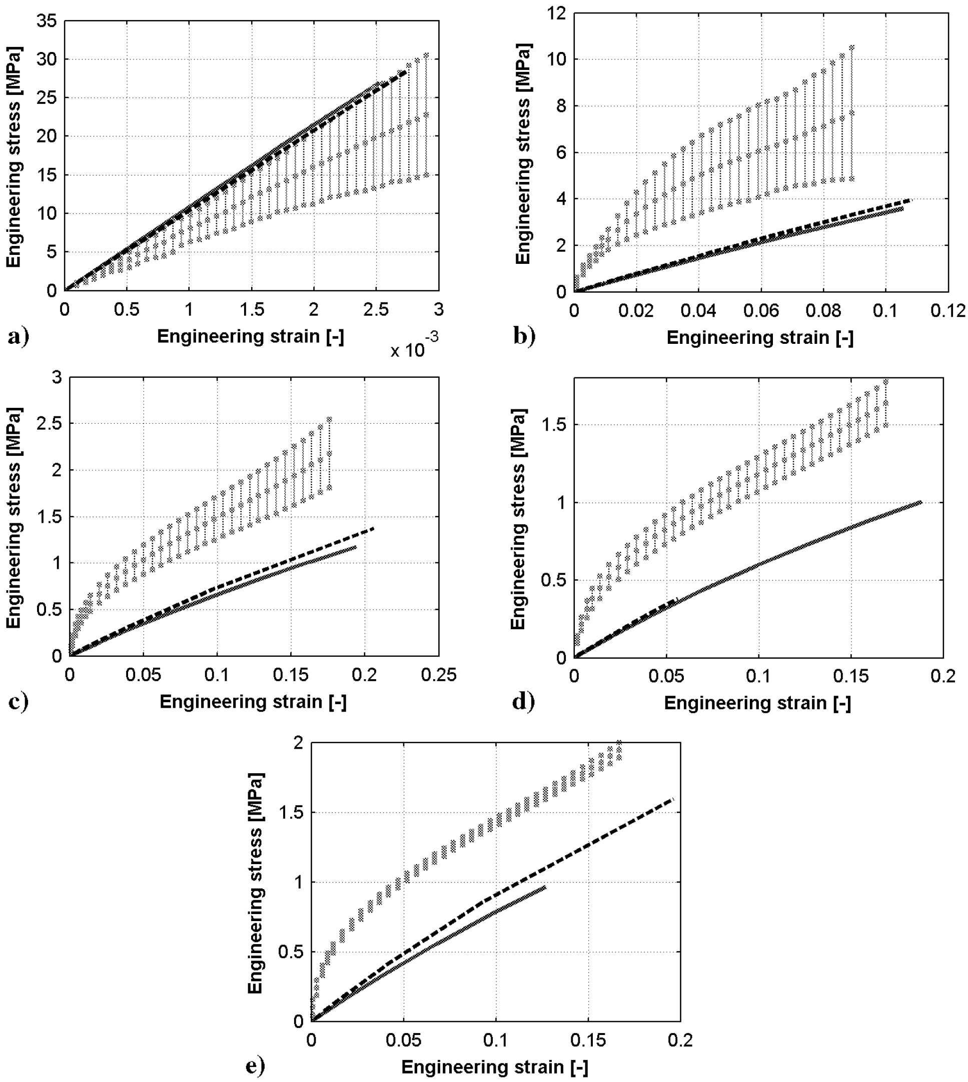

Fig.6: Results of the tension tests and their simulations with various declinations of fibres: a) 0°, b) 15°, c) 45°, d) 60°, e) 90°.

amplitude. The specimens made from composite material were also preconditioned, but in a different way. Each specimen was loaded by 5 mm of total elongation. This elongation generates various strains in the specimens depending on the declination angle of fibres. Next, strains and stresses in the specimens are not homogenous and each part of the matrix is loaded by various strains. For these reason, it is not possible to perform an experiment, where the whole specimen will be preconditioned by the same strain amplitude. Different amplitudes in precycling lead to different strain-stress curves (due to the Mullin's effect [11]) and therefore the simulations cannot give the same results as the experiments. A feasible solution can be application of constitutive models being able to work with Mullin's effect including different amplitude of loads, e.g. Ogden-Roxburg's model [12]. However, none of the known FEM systems contains an anisotropic hyperelastic model with a description of Mullin's effect.

The next five figures (Fig. 7a) to e)) show results from the experiments and simulations of bending tests.

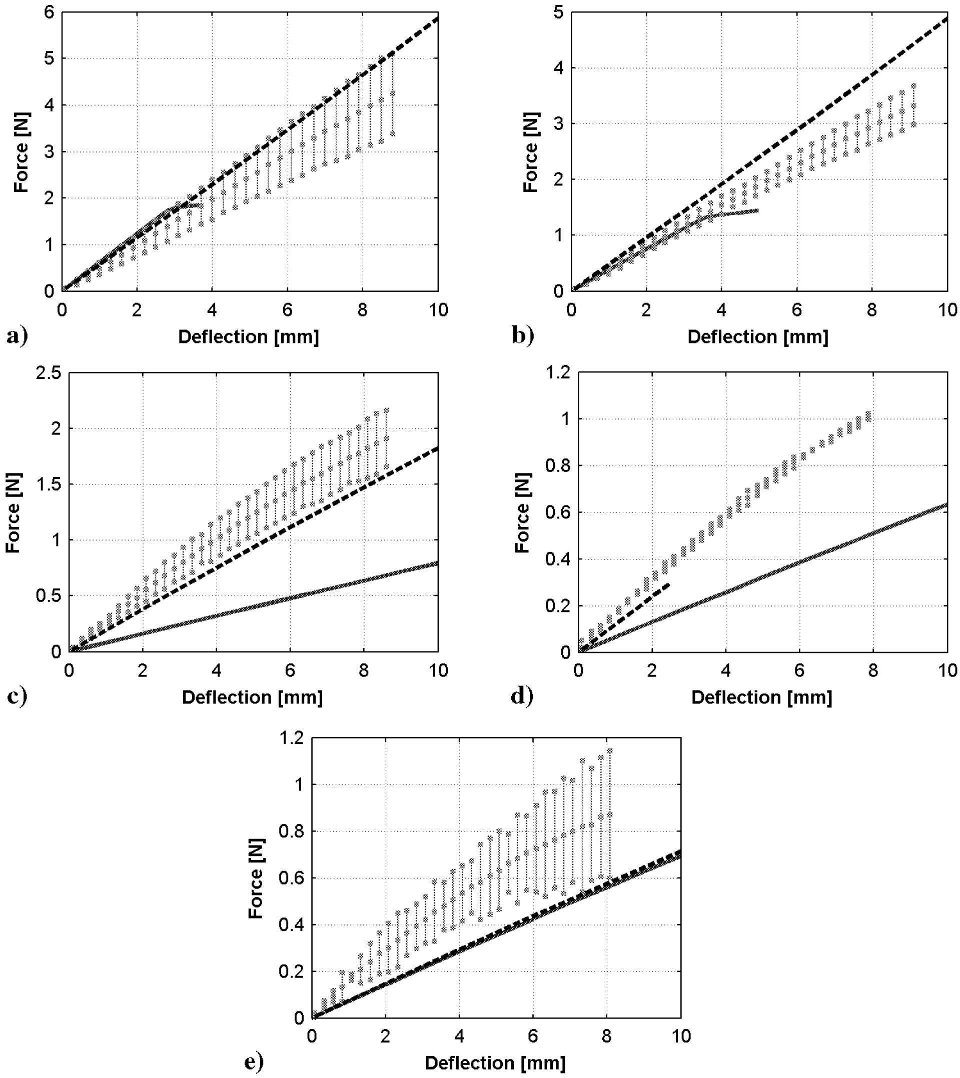

Fig.7: Results of the bending tests with various declinations of fibres : a) 0°, b) 15°, c) 45°, d) 60°, e) 90°.

For an easy orientation the same legend was used as in the case of tension tests. The results show that both unimaterial and bimaterial models disagree with the experiments. We supposed that this disagreement is caused by the same Mullin's effect as in the case of the tension tests; hence experiments with simulations which were performed under various preconditioning values should not be compared. For illustration of the influence of Mullin's effect, bending tests of the pure matrix and composite material with the fibre declination of 90 degrees are depicted in Fig. 8. It is evident that both curves should be at least the same or the curve obtained from bending of the composite should be rather stiffer, because fibres prohibit a transverse contraction of the matrix. However, we can see that bending of the pure matrix (dark curve) is stiffer than bending of composite (light curve). The explanation of this discrepancy can be that the matrix itself was preconditioned by 100 % strain, while the composite was preconditioned by 200 % strain.

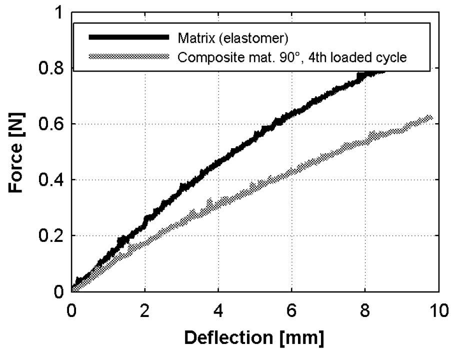

Fig.8: Influence of Mullin's effect

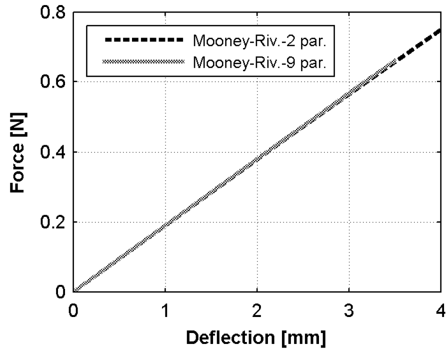

Fig.9: Different constitutive models of matrix

Next, we can compare the results obtained by both models. In Fig. 7a) we can see a particular behaviour of the unimaterial model. Within deflection of approximately 3 mm, the unimaterial model is in very good agreement with the bimaterial one; nevertheless, with the increasing deflection of the specimen a substantial disagreement occurs. Similar behaviour can be observed in Fig. 7b). In the next two figures 7c) and 7d), a substantial disagreement between the models can be seen, while both models are in very good agreement in Fig. 7e).

The anisotropic hyperelastic constitutive model (3) or (4) is based on the assumption that the fibres are infinitesimally thin, i.e. they are ideally flexible (zero bending stiffness of an individual fibre). Hence, in the case of fibres under 0° or 15°, buckling of the fibres occurs at some deflection value. This state is the most energy advantageous (fibres have zero bending stiffness). Essentially, the fibre is compressed from the beginning of the loading process (or a part of the fibre being above the neutral plane), but buckling of fibres doesn't occur due to the supporting effect of the matrix. However, the matrix is not more able to prevent buckling after some deflection value and this can result in turning points of the curves in Fig. 7a) or 7b) (unimaterial model); consequently, the numerical solution brakes down. On the contrary, in the composite material with the fibre declination of 90 degrees, the fibres do not increase the stiffness of the composite material substantially; essentially the bending occurs in the pure elastomer matrix and the fibres prohibit only the transverse contraction of matrix. Hence, both models give the same results.

For illustration, results of simulations with two different constitutive models for the hyperelastic matrix are shown in Fig. 9. The results are for the bending test with fibre declination of 45 degrees, and the simulations were performed with the bimaterial computational model. It is evident, that constitutive model with higher terms (9-parameter Mooney-Rivlin) gives nearly the same results as the model with two terms only (2-parameter Mooney-Rivlin). It can be concluded, that a more precious constitutive model of the matrix cannot contribute to the improvement of the results.

A feasible solution of this disagreement can be found in addition of the bending stiffness into the anisotropic hyperelastic model. Spencer [10] assumed that the strain energy density function of the anisotropic hyperelastic model depends on Cauchy-Green deformation tensor, fibre direction and **newly on the gradients of the deformed fibre vectors**. Dependence on the gradient of the deformed fibre vectors incorporate bending stiffness into the constitutive model mentioned in [1, 2]. Constitutive equations for this last computational model are introduced in [10], where it is shown that incorporating the bending stiffness by means of the gradients results in the Cosserat theory of elasticity [8, 9].

Unfortunately, with regard to the fact that application of Cosserat theory in description of anisotropic hyperelastic materials is quite a new approach (as documented by date of publication of [10] – as late as 2007), there are still no publications about its practical application in this field, which could be able to confirm or reject the expected advantages of Cosserat theory.

5. Conclusion

In the paper two types of simulations of mechanical tests, performed at various levels of computational models, were compared with each other and with the experiments on a composite material with a hyperelastic matrix and steel fibres. For this purpose the experiments and simulations of tension and bending tests were carried out; then two computational models of various levels were used for the simulations. The first one was the bimaterial model, which includes 3D geometry models of both matrix and fibres. The second one was the unimaterial model, in which the fibre and matrix materials are not distinguished, but the reinforcement effect of fibres is described by the anisotropic hyperelastic constitutive model with polynomial strain energy density function.

The simulations showed that both models give nearly the same results in case of tension tests, while in case of bending tests the results differ substantially. The reason of this discrepancy is that the unimaterial model assumes the fibres to have a zero bending stiffness. This problem could be solved feasibly by introducing the bending stiffness into equations of an anisotropic hyperelastic model. This was already done by Spencer, who also derived new constitutive equations. These equations, based on Cosserat theory of elasticity, were not published but quite recently (2007) and they are highly complex. This may be the reason that there are no publications about their practical application till now.

The presented computational simulations showed that the unimaterial model can be used in case of tension (compression) load instead of the bimaterial one, while in case of bending load the unimaterial model is not able to include bending stiffness of the fibres. Hence, the unimaterial model is suitable only for applications where tension or compression of the fibres predominate.

Next, it was find out the results of experiments depend strongly on the amplitude of preconditioning (Mullin's effect). The strain-stress curve becomes softer with the increasing preconditioning amplitude. Since the strain and stress states in the composite specimen are not homogenous, it is not possible to perform an experiment, where the whole composite specimen with a non-trivial arrangement will be cycled with the same amplitude. In each point of the specimen, hence, the matrix material follows another stress-strain curve and none of such simulations can give results comparable with the experiments. To avoid this discrepancy, the constitutive model used in simulations must be able to include Mullin's effect with various strain amplitudes. At present, however, no such models are implemented in any FEM software together with an anisotropic hyperelastic model.

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